

POLARIZATION-ENTANGLED PHOTON GENERATION BY A SEMICONDUCTOR QUANTUM DOT COUPLED TO A CAVITY INTERACTING WITH EXTERNAL FIELDS

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ABSTRACT. We theoretically investigate polarization-entangled photon generation by using a semiconductor quantum dot embedded in a microcavity. The entangled states can be produced by the application of two cross-circularly polarized laser fields. The quantum dot nanostructure is considered as a four-level system (ground, two excitons and bi-exciton states) and the theoretical study relies on the dressed states scheme. The quantum correlations, reported in terms of the entanglement of formation, are extensively studied for several values of the important parameters of the quantum dot system as the bi-exciton binding energy, the decoherence times of the characteristic transitions, the quality factor of the cavity and the intensities of the applied fields.

1. INTRODUCTION

In the relatively new scientific field of quantum information and quantum computing [1], the creation and control of entangled-photon pair is a rather key issue [2, 3, 4, 5]. The most used (common) way of generating entangled-photon pairs is through a parametric down conversion [6, 7]. An alternative technique for entangled-photon generation is the two-photon cascade decay of atomic excitations [8, 9, 10]. Recently, an extension of the atomic technique, the cascade-emission process from a biexciton state in a semiconductor quantum dot (SQD, usually called artificial atoms), was proposed [11] and demonstrated [12, 13, 14, 15]. In this more efficient entangled-photon generation scheme, the entangled photons are generated from the cascade emission through the degenerate intermediate states having different polarizations.

A much better control of this process is achieved by placing the SQD in a microcavity, where the material excitations and the photons of the cavity are strongly coupled. Then the SQD-cavity system is described in terms of the coupled states of material excitations and cavity photons, which usually called dressed states (properly treated using cavity quantum electrodynamics, i.e. describing matter and light on quantum mechanical grounds). Actually, it has been shown in recent theoretical investigations that under specific optimal conditions a drastic enhancement of the entangled-photon generation can be achieved for SQDs embedded in a cavity [16, 17, 18, 19]. This is achieved, as the dressed-states formed in the SQD-cavity system eliminate the which-path information which prevents the formation of entangled-photon pair in the biexciton decay process. Moreover, the energies of these intermediate dressed states are tuned to be degenerate states by using a high-Q cavity [20, 21, 22].

Recently the entangled-photon generation from a SQD-cavity system, where the biexciton is efficiently excited under resonant conditions, was studied in Refs. [16, 17, 18, 19].

The resonant excitations of biexciton occur through the dressed biexciton states. Two of the four Bell states can be generating by selecting the frequencies of applied fields with certain polarizations (cross-circularly polarized fields) [16, 19]. The other two Bell states can be generated by changing the combination of polarizations or by properly adjusting only the frequencies of the input fields[16, 18]. Furthermore, they have achieved control of the non-entangled co-polarized photons [19], as they are strongly suppressed due to the photon blockade effect [23, 24, 25].

In the present paper we investigate theoretically the entangled - photon generation from an SQD-cavity system by calculating the thermal entanglement of Formation (EoF). This paper is organized as follows. In Sec. II, we describe our model of the QD-cavity system and the resulting dressed states. The Bell states generated from the dressed states are presented in Sec. III. In Sec. IV, we present numerical results on the entangled-photon pairs from the QD-cavity system in the presence of a biexciton. Our results are summarized in Sec.V.

2. THEORY

We describe the QD nanostructure by a four-level system taking into account the ground state $|G\rangle$, the two circularly right and left polarized degenerate exciton states $|X_R\rangle, |X_L\rangle$ and the biexciton state $|B\rangle$, which are orthogonal and normalized. In addition we consider that the QD is embedded in a microcavity which supports circularly right- and left- polarized photons of energy equal to the energy difference between the exciton and the ground state. Actually, we should be careful if we have to include other (excited) exciton states and biexciton states. The present model is valid once the level separation of excitons and biexcitons are greater than the vacuum Rabi splitting. These approximations are valid in the GaAs SQDs as for realistic values of the later dimensions of the SQD the rabi splitting is of the order of few tenths of meV, while the level separation is a few meV [26]. Then, the QD-cavity system is represented by a Hilbert space which is the direct product of states of the QD nanostructure and the cavity-mode photons. Hence the representation of the product states has the following form, $|Y, n_R, n_L\rangle$ where Y describes the energy states of the QD and $n_R(n_L)$ specifies the number of the right- (left-) polarized photons in the cavity. In fact the presence of an anisotropic electron-hole exchange interaction results in the well known fine structure splitting (FSS) of the two excitonic states. However in our study we assume that the two exciton states are degenerate, assuming that the FSS has been minimized by either properly treating the QD structural properties, as for example by controlling the growth process, or by applying external (mechanical, electric and magnetic) fields. The Hamiltonian describing the QD-cavity quantum system is given by

$$\begin{aligned}
 H_0 = & \epsilon_0 \sum_k (|X_k\rangle\langle X_k| + \alpha_k^\dagger \alpha_k) \\
 & + (2\epsilon_0 - \Delta_B) |B\rangle\langle B| \\
 & + g \sum_k (i|G\rangle\langle X_k| \alpha_k^\dagger + H.c.) \\
 & + g_B (i|X_R\rangle\langle B| \alpha_R^\dagger + i|X_L\rangle\langle B| \alpha_L^\dagger + H.c.)
 \end{aligned}
 \tag{1}$$

where the index k runs over the R (right- polarized) and L (left-polarized) QD states or cavity-mode photons. ϵ_0 is the energy difference between the exciton and the ground state. $g(g_B)$ is the coupling constants between the exciton(biexciton) transition and one cavity-mode photon. Δ_B is the binding energy of the biexciton state. The ground state has zero

energy. In this article we adopt the natural units ($\hbar=1$). It is rather straightforward to diagonalize the H_0 Hamiltonian and find its eigenstates, usually called dressed states. These dressed states being a linear superposition of product states are characterized and grouped by the total number of QD excitations and photons. The product states are divided into one state of zero energy ($|G, 0, 0\rangle$), four states of ϵ_0 energy ($|G, 1, 0\rangle$, $|G, 0, 1\rangle$, $|X_R, 0, 0\rangle$ and $|X_L, 0, 0\rangle$), seven states of $2\epsilon_0$ energy ($|G, 1, 1\rangle$, $|G, 2, 0\rangle$, $|G, 0, 2\rangle$, $|X_R, 1, 0\rangle$, $|X_R, 0, 1\rangle$, $|X_L, 1, 0\rangle$ and $|X_L, 0, 1\rangle$) and one state of $2\epsilon_0 - \Delta_B$ energy ($|B, 0, 0\rangle$). The dressed states are categorized into three groups. The group with the lowest (zero) energy which is actually the $|G, 0, 0\rangle$ product state. The second group with states of higher energy, all in the region of ϵ_0 . Actually we have only two energy values with a $2g$ energy splitting known as the vacuum Rabi splitting. These dressed states are a superposition of singly excited QD of specific polarization and absence of photons and ground state QD in the presence of one photon with the same polarization and hence can be characterized by their polarization (namely, $|R_+\rangle$, $|R_-\rangle$, $|L_+\rangle$, and $|L_-\rangle$). Finally, we have eight dressed states of energy around $2\epsilon_0$. We can separate these states into two categories, the co-polarized dressed states (linear superposition of $|G, 2, 0\rangle$, $|G, 0, 2\rangle$, $|X_R, 1, 0\rangle$, and $|X_L, 0, 1\rangle$) denoted as $|RR_+\rangle$, $|RR_-\rangle$, $|LL_+\rangle$ and $|LL_-\rangle$ and the cross-polarized dressed states (linear superposition of $|G, 1, 1\rangle$, $|B, 0, 0\rangle$, $|X_R, 0, 1\rangle$, $|X_L, 1, 0\rangle$) corresponding to a singlet state

$$|S\rangle = \frac{1}{\sqrt{2}}(|X_R, 0, 1\rangle - |X_L, 1, 0\rangle)$$

of energy $2\epsilon_0$ and three triplet states. Additionally, the cross-polarized states have eigenenergies of the form $\lambda_j = 2\epsilon_0 - a_j$, where $j = 1, 2, 3$ [19]. Substantially, a_j are the differences of $2\epsilon_0$ from each λ_j and they are the real solutions, multiplied by Δ_B , of the equation [19],

$$(2) \quad f(x) = x^3 - x^2 - (p + q)x + p = 0$$

with $p = 2(\frac{g}{\Delta_B})^2$ and $q = 2(\frac{g_B}{\Delta_B})^2$.

In general the cavity is coupled to the environment by cavity-mode photons leaking out of the cavity. In order to have a control of these leaking photons we apply external electromagnetic fields interacting with the microcavity. We consider CW laser fields with electric fields $E_R e^{-i\Omega_R t}$ and $E_L e^{-i\Omega_L t}$, where E_R (E_L) and Ω_R (Ω_L) are the amplitudes and the frequencies of the right-(left-) polarized laser fields all assumed real. Then the Hamiltonian describing the interaction between the lasers and the cavity photons is [19, 27]

$$(3) \quad H_{int} = \sqrt{\Gamma} \sum_{k=R,L} E_k (ie^{-i\Omega_k t} \alpha_k^\dagger - ie^{i\Omega_k t} \alpha_k)$$

where Γ is a phenomenological parameter describing the cavity photon leakage. The dynamics of the density matrix operator of the whole quantum system ($H = H_0 + H_{int}$) in which the damping of the excited states is taken into account is described by means of the following standard master equation

$$(4) \quad \begin{aligned} \frac{d}{dt} \rho(t) = & -i[H, \rho(t)] \\ & + \gamma_X \left(\frac{1}{2} \{ |G\rangle, |X_R\rangle \} \rho(t) + \frac{1}{2} \{ |G\rangle, |X_L\rangle \} \rho(t) \right) \\ & + \gamma_B \left(\frac{1}{2} \{ |X_R\rangle, |B\rangle \} \rho(t) + \frac{1}{2} \{ |X_L\rangle, |B\rangle \} \rho(t) \right) \\ & + \Gamma \{ \mathbb{1}, \alpha_k^\dagger \} \rho(t) \end{aligned}$$

where the operator

$$\{u, v\}_f = 2uu^\dagger v^\dagger f v - f v v^\dagger - v v^\dagger f$$

and γ_X , γ_B are the damping constants from the exciton and biexciton, respectively. The last term of eq.(4) represents the photon leakage from the cavity to the environment.

We focus on the steady state solution of the master equation achieved, at sufficiently large t , by vanishing the time derivative, of the coupled non-linear differential equations of the master equation and practically solving an algebraic system of non-linear equations.

3. ENTANGLED PHOTON GENERATION

We work in the weak coupling regime assuming all the coupling constants much smaller than the characteristic energy, ϵ_0 of the system under investigation. Moreover, we consider the weak fields limit of very small amplitudes of both the external electric fields. Both conditions should be met as under these conditions eq.(4) is a valid master equation. Now, we turn our attention to the emitted photon of the QD-cavity system. As we are in the weak field limit, it is sufficient to consider only the subspaces of zero, one and two excitations of QD and cavity simultaneously. Actually it is more convenient to work with the dressed states of the QD-cavity system and assume only cascade transitions between neighboring subspaces. Hence the corresponding transitions and emissions of a k-polarized photons are represented by the following operators

$$(5) \quad (|n\rangle\langle n|)\alpha_k(|m\rangle\langle m|) = |n\rangle(\langle n|\alpha_k|m\rangle)\langle m| = \gamma_{n,m}|n\rangle\langle m|,$$

where $\gamma_{n,m}$ is the transition amplitude from the $|n\rangle$ to the $|m\rangle$ ($\gamma_{n,m}$ are summarized in Ref.[19], from eq.(15) till eq.(19)). As the energy splitting of the dressed states in the two upper subsets is unequal we can take advantage of this and create entangled photons. The idea is the following. We use input field frequencies Ω_L or Ω_R tuned to the lower state of single excitation (of energy $\epsilon_0 - g$). Then the photon pairs generated via the upper state of single excitation (of energy $\epsilon_0 + g$) would have energies which are different from Ω_L and Ω_R . Then by performing spectral filtering we extract these photons. Hence, one possible scheme would be to tune Ω_L to the $\epsilon_0 - g$ state resonantly driving the $|G\rangle \rightarrow |L_-\rangle$ transition, and consider spectral filtering of the $(\Omega_R + \Omega_L) - (\epsilon_0 + g)$ and $\epsilon_0 + g$. In order to estimate any property of the photon pairs generated via the cascade process we need the reduced density matrix where the bases are in the following order $|L(\omega_1)\rangle|R(\omega_2)\rangle$, $|R(\omega_1)\rangle|L(\omega_2)\rangle$, $|L(\omega_1)\rangle|L(\omega_2)\rangle$ and $|R(\omega_1)\rangle|R(\omega_2)\rangle$, with photon frequencies $\omega_1 = (\epsilon_0 - g)$ and $\omega_2 = \Omega_R + \Omega_L - (\epsilon_0 - g)$.

Hence our major task is to find the density matrix, $\rho_{\text{ph-pairs}}$ of this four dimensional ($|L\rangle|R\rangle$, $|R\rangle|L\rangle$, $|L\rangle|L\rangle$, $|R\rangle|R\rangle$) Hilbert space, as once we know the $\rho_{\text{ph-pairs}}$ density matrix we can extract any information for the classical and quantum correlations of the photon pairs generated in the QD-cavity system. The $\rho_{\text{ph-pairs}}$ density matrix can be calculated by properly tracing out the ρ density matrix describing the whole QD-cavity system. As the photon pairs $|nm\rangle$ are generated through cascade transitions through the $|R+\rangle$ or $|L+\rangle$ dressed states, we can describe this process by transition operators, $T(nm)$ acting on the total Hamiltonian, H . For example the transition operator which creates the $|RR\rangle$ pair is given by

$$(6) \quad \begin{aligned} T(RR) &= \gamma_{G;R+} |G\rangle\langle R+| \sum_{k=-,+} \gamma_{R+;RRk} |R+\rangle\langle RRk| \\ &= \gamma_{G;R+} \sum_{k=-,+} \gamma_{R+;RRk} |G\rangle\langle RRk| \end{aligned}$$

The density matrix, $\rho_{\text{ph-pairs}}$ can be represented as

$$(7) \quad \rho_{\text{ph-pairs}} = \sum_{n', m', n, m=R, L} (\langle n' m' | \langle nm |) | n' m' \rangle \langle nm |$$

where the coefficients $\langle n' m' | \langle nm |$ are proportional to the $\text{Tr}[\rho T^+(n' m') T(nm)]$. The proportionality coefficients are determined from the condition $\text{Tr}[\rho_{\text{ph-pairs}}] = 1$.

4. NUMERICAL RESULTS AND DISCUSSION

In this section, we report the degree of entanglement (EoF) of the photon pairs generated from the QD-cavity system for different values of system parameters. Figure.1 shows EoF as a function of g and $\Omega'_R = \Omega_R - \epsilon_0$ for fixed values of $g_B = 15\Gamma$ and the input field frequency at $\Omega_L = \epsilon_0 - g$ (i.e. tuned at the lowest possible transition) and with three different values of the biexciton binding energy. A relatively low value of Δ_B of 7Γ (e.g. InAs QD) at fig.1(a), an intermediate value of 15Γ (e.g. GaAs/AlAs QD) at fig.1(b) and a high one of 150Γ (e.g. CuCl QD) at fig.1(c).

In all figures ((a) to (c)) it is apparent that the largest values of the entanglement reported by EoF occur along the lines which correspond to resonant transitions of the upper four dressed states to the intermediate state (of energy $\epsilon_0 + g$). The four major lines denote resonant excitations for the dressed state $|T_1\rangle$ (top curve with positive slope), $|S\rangle$ (second curve), $|T_2\rangle$ (third curve) and $|T_3\rangle$ (bottom curve with positive slope). There are two more lines denote resonant excitations for the dressed state $|RR_-\rangle$ (curve with negative slope between the intermediate single and triple curves) and $|RR_+\rangle$ (bottom curve with negative slope in plots (a) and (b)). For the high value of the binding energy (plot (c)) the two latter curves disappear or one changes slope. In the low and intermediate binding energies (plots (a) and (b)), we find that there is a gap where the EoF becomes zero. This disappearance of the entanglement originates by the fact that the transition amplitude $\gamma_{L+;T_1}$ becomes zero. Under this condition we obtain that for $g = g_-$, with $g_- = \frac{1}{4}(\sqrt{\Delta_B^2 + 16g_B^2} - \Delta_B)$ [19]. As expected, when we increased the binding energy this gap moved towards to the beginning of the excitation line of $|T_1\rangle$. Moreover, for small values of Δ_B the dressed states $|T_2\rangle$ and $|S\rangle$ are almost degenerated but in our case this is not true. Indeed, there are two regions of maximum entanglement at the center of the diagram which means that the two dressed states are not degenerated. This happens, as from [19] we know the relation $a_1 < 0 < a_2 < \Delta_B < a_3$. As a result, for small values of the binding energy Δ_B , the value of a_2 is almost zero, thus the dressed states $|S\rangle$ and $|T_2\rangle$ are almost degenerate, which is not happen in our case as mentioned above.

By systematically investigating cases of binding energy higher than the one of 150Γ (figure1(c)), we have found that the EoF diagram remains practically unchanged. The explanation of the latter behaviour is given by means of figure 2 where we reveal that the dressed states energies (fig. 2(a)) and the transition rates (fig. 2(b)) remain unchanged above an binding energy at 160Γ .

Figure 3 shows the entanglement of formation (EoF) for a wider range of g and Ω'_R . Now we have also set $\gamma_X = \gamma_B = 0.1\Gamma$. A thin line of maximum EoF surrounded by a region where the entanglement is zero or very small, is observed at the center of the diagram between of excitation lines of $|S\rangle$ and $|T_2\rangle$. Additionally, we observe other thin lines of zero entanglement just under of the above line at the center of the diagram too, as well as under the line of excitation with the dressed state $|T_3\rangle$. Actually under the line of $|T_3\rangle$ there are two thin lines of zero entanglement. From [19] we know the relations $a_1 < 0 < a_2 < \Delta_B < a_3$ as well as $a_1 < -\sqrt{2}g < a_2 < \sqrt{2}g < a_3$, where the a_j is the difference

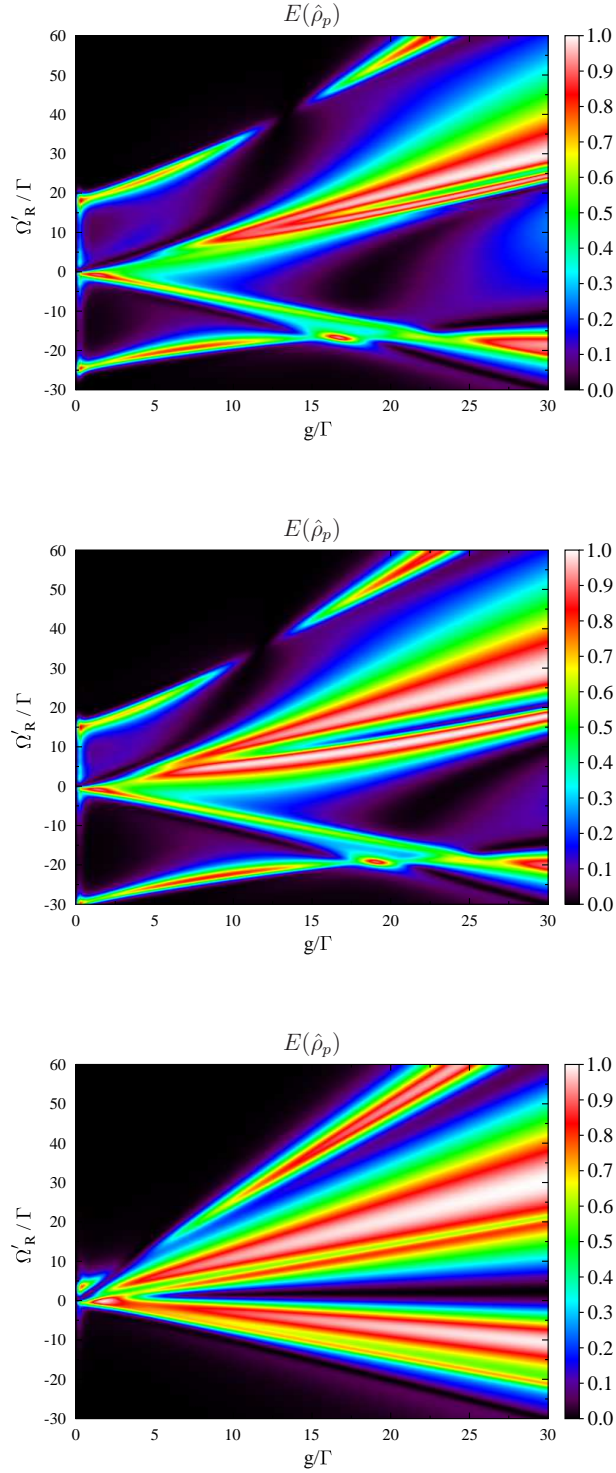


FIGURE 1. The degree of entanglement (EoF) for the QD-cavity system as a function of the g/Γ for (a) $\Delta_B = 7\Gamma$, (b) $\Delta_B = 15\Gamma$ and (c) $\Delta_B = 150\Gamma$. The highest degree of entanglement, for all cases, is observed at the center of the diagram and more precisely for the dressed states $|S\rangle$ and $|T_2\rangle$.

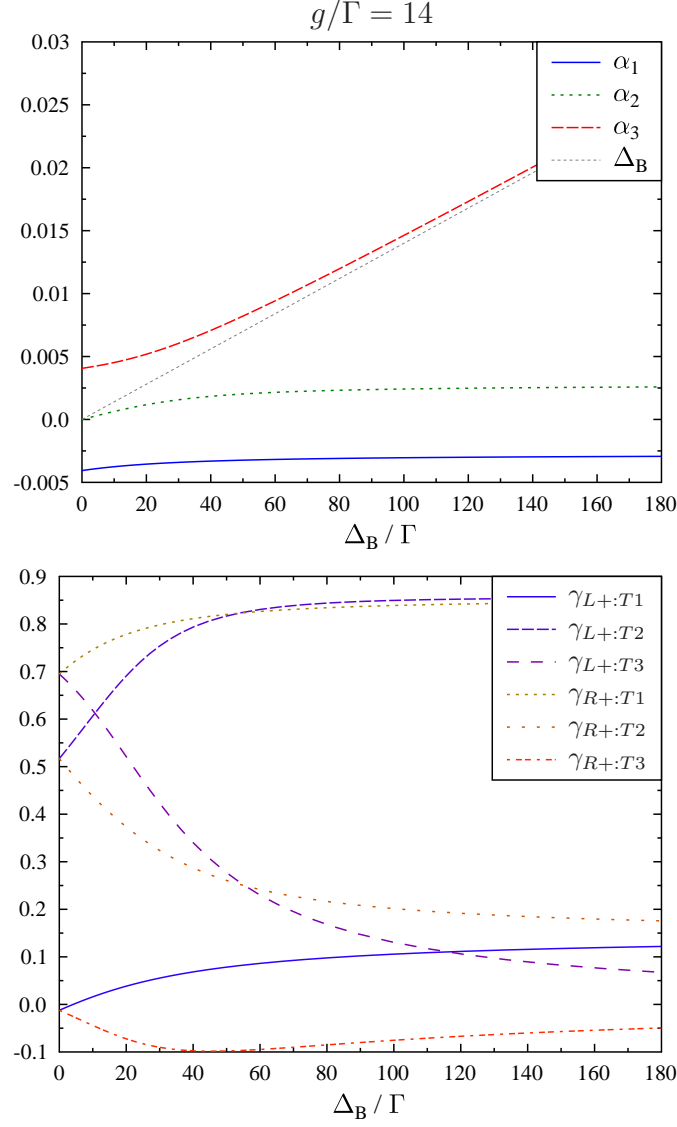


FIGURE 2. (a) The energy of the dressed T-states and (b) the transition amplitudes of the dressed T-states to $R+$ and $L+$ states as a function of the biexciton binding energy.

between $2\omega_0$ and the corresponding eigen-energies. By adding these relations we also have $a_1 < -g/\sqrt{2} < a_2 < g/\sqrt{2} + \Delta_B/2 < a_3$. These relations indicate the possible values but also give information about the forbidden values of a_j . More specifically, it is evident that a_j cannot be zero for example as must be greater or smaller than this value. As a consequence, there are forbidden areas in this figure where the entanglement becomes zero. In one of these lines, as mentioned above, there is a thin line of maximum. This happens because for $\Omega'_R = g - \Delta_B$ there is a forbidden area but also there is resonance with the dressed state $|T_2\rangle$ with $\Omega'_R = g - a_2$, with $0 < a_2 < \Delta_B$.

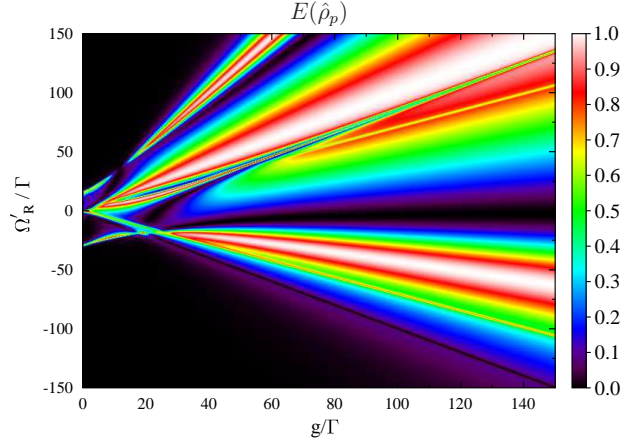


FIGURE 3. The degree of entanglement (EoF) for the QD-cavity system as a function of the g/Γ for $\Delta_B = 15\Gamma$, $g_B = 15\Gamma$, $\chi_X = \gamma_B = 0.1\Gamma$, $E_R = E_L = 0.02\sqrt{\Gamma}$.

Finally, we systematically investigate the effect of the laser intensities. Figure 4(a) shows the degree of entanglement as a function of g and Ω'_R with $E_R = 0.01\sqrt{\Gamma}$ and $E_L = 0.02\sqrt{\Gamma}$ whereas in figure 4(b) is $E_R = 0.02\sqrt{\Gamma}$ and $E_L = 0.01\sqrt{\Gamma}$ and all the other parameters are exactly the same. As it is easily observable the two diagrams are not symmetric to each other. This is because the left-polarized laser field is tuned to the state $|L_- \rangle$ as we have $\Omega_L = \omega_0 - g$, thus we have more excitations and as a consequence more generated entangled photon pairs in the second case. For this reason, in figure 4(b) we have many regions with high degree of entanglement. We observe maximum EoF along the lines which indicate resonant with the dressed states $|S \rangle$, $|T_j \rangle$ (with $j=1,2,3$) as well as the co-polarized state $|RR_- \rangle$.

It is worth noting that the gap where the EoF becomes zero across the line of $|T_1 \rangle$ still remains but this is not true for the maximum along the line of $|RR_- \rangle$ where instead of a maximum we observe a minimum. This is because the pairs which generated from the state $|RR_- \rangle$ or $|LL_- \rangle$ are not entangled any more since we use lasers with different amplitudes thus, the probability to have excitation with a right-polarized photon is greater than the corresponding probability of an excitation with a left-polarized photon. Additionally there is a thin line of zero entanglement between the excitation lines of $|S \rangle$ and $|T_2 \rangle$. On the other hand, in figure 4 there are only two maximums along the excitation lines of $|S \rangle$ and $|T_2 \rangle$. Obviously, the results will be reversed if one tunes Ω_L laser frequency to the $\epsilon_0 + g$ state resonantly driving the $|G \rangle \rightarrow |L_+ \rangle$ transition, and consider spectral filtering of the $(\Omega_R + \Omega_L) - (\epsilon_0 + g)$ and $\epsilon_0 - g$.

5. CONCLUSIONS

In summary, in the present work, we theoretically investigated the generation of polarization-entangled photons from a microcavity in which a QD is embedded in, radiated by two laser fields of opposite circular polarization (left/right). Our theoretical model relies on the

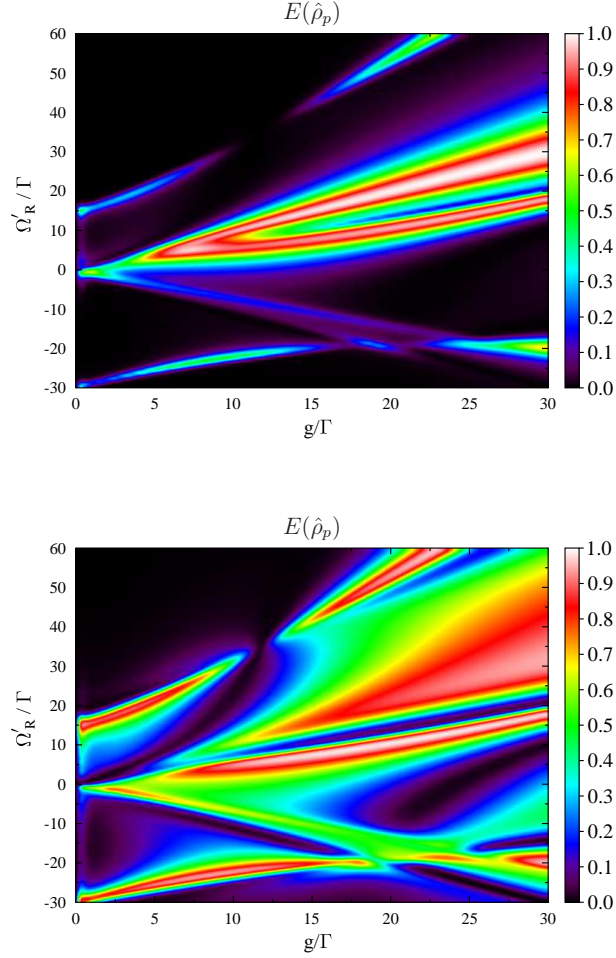


FIGURE 4. Entanglement of formation (EoF) for (a) $E_R = 0.01\sqrt{\Gamma}$ and $E_L = 0.02\sqrt{\Gamma}$. There are maximums only for resonant with $|S\rangle$ and $|T_2\rangle$ and for (b) $E_L = 0.01\sqrt{\Gamma}$ and $E_R = 0.02\sqrt{\Gamma}$. The entanglement has completely different behaviour in comparison to the previous case (a). Although should be noted that in both cases only two Bell-states, $|B_1\rangle = \frac{1}{\sqrt{2}}(|RL\rangle + |LR\rangle)$ and $|B_2\rangle = \frac{1}{\sqrt{2}}(|RL\rangle - |LR\rangle)$ are generated.

dressed states scheme of the QD nanostructure considered as a four-level system and containing energies up to the order of the two exciton states). By systematically investigate the entanglement of formation, for various values of the important parameters of the quantum dot system as the bi-exciton binding energy and the intensities of the applied fields we have shown that there is a strong dependence of the quantum correlations on them.

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REFERENCES

- [1] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
- [2] A. Barenco, D. Deutsch, A. Ekert, and R. Jozsa, Phys. Rev. Lett. 74, 4083 (1995).
- [3] T. Sleator and H. Weinfurter, Phys. Rev. Lett. 74, 4087 (1995).
- [4] J. I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4091 (1995).
- [5] Anton Zeilinger, *Dance of the photons: from Einstein to quantum teleportation* (Farrar, Straus and Giroux, New York, 2010).
- [6] P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, Phys. Rev. Lett. 75, 4337 (1995).
- [7] P. G. Kwiat, E. Waks, A. G. White, I. Appelbaum, and P. H. Eberhard, Phys. Rev. A 60, R773 (1999).
- [8] C.A. Kocher and E.D. Commins, Phys. Rev. Lett. 18 575 (1967).
- [9] J.F. Clauser, Phys. Rev. Lett., 36 1223 (1976).
- [10] A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. 47 460 91981).
- [11] O. Benson, C. Santori, M. Pelton, and Y. Yamamoto, Phys. Rev. Lett. 84, 2513 (2000).
- [12] N. Akopian, N. H. Lindner, E. Poem, Y. Berlatzky, J. Avron, D. Gershoni, B. D. Gerardot, and P. M. Petroff, Phys. Rev. Lett. 96, 130501 (2006).
- [13] R. Young, R. Stevenson, P. Atkinson, K. Cooper, D. Ritchie, and A. Shields, New J. Phys. 8, 29 (2006).
- [14] R. Hafenbrak, S. Ulrich, P. Michler, L. Wang, A. Rastelli, and O. Schmidt, New J. Phys. 9, 315 (2007).
- [15] A. Dousse, J. Suffczynski, A. Beveratos, O. Krebs, A. Lematre, I. Sagnes, J. Bloch, P. Voisin, and P. Senellart, Nature (London) 466, 217 (2010).
- [16] H. Ajiki and H. Ishihara, Phys. Status Solidi C 6, 276 (2009).
- [17] H. Ajiki, H. Ishihara, and K. Edamatsu, New J. Phys. 11, 033033 (2009).
- [18] H. Ajiki, H. Ishihara, and K. Edamatsu, Physica E 40, 371 (2007).
- [19] K. Shibata and H. Ajiki, Phys. Rev. A 86, 032301 (2012).
- [20] R. Johne, N. A. Gippius, G. Pavlovic, D. D. Solnyshkov, I. A. Shelykh, and G. Malpuech, Phys. Rev. Lett. 100, 240404 (2008).
- [21] R. Johne, N. A. Gippius, and G. Malpuech, Phys. Rev. B 79, 155317 (2009).
- [22] P. K. Pathak and S. Hughes, Phys. Rev. B 79, 205416 (2009).
- [23] A. Imamoglu, H. Schmidt, G. Woods, and M. Deutsch, Phys. Rev. Lett. 79, 1467 (1997).
- [24] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble, Nature (London) 436, 87 (2005).
- [25] A. Faraon, I. Fushman, D. Englund, N. Stoltz, P. Petroff, and J. Vuckovic, Nat. Phys. 4, 859 (2008).
- [26] D. Gammon, E.S. Snow, B.V. Shanabrook, D.S. Katzer and D. Park, Phys. Rev. Lett. 76, 3005 (1996).
- [27] H. Ajiki, J. Opt. B - Quant. Sem. Opts. 7, 29 (2005).

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